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MISSILE ATTITUDE COMPUTER
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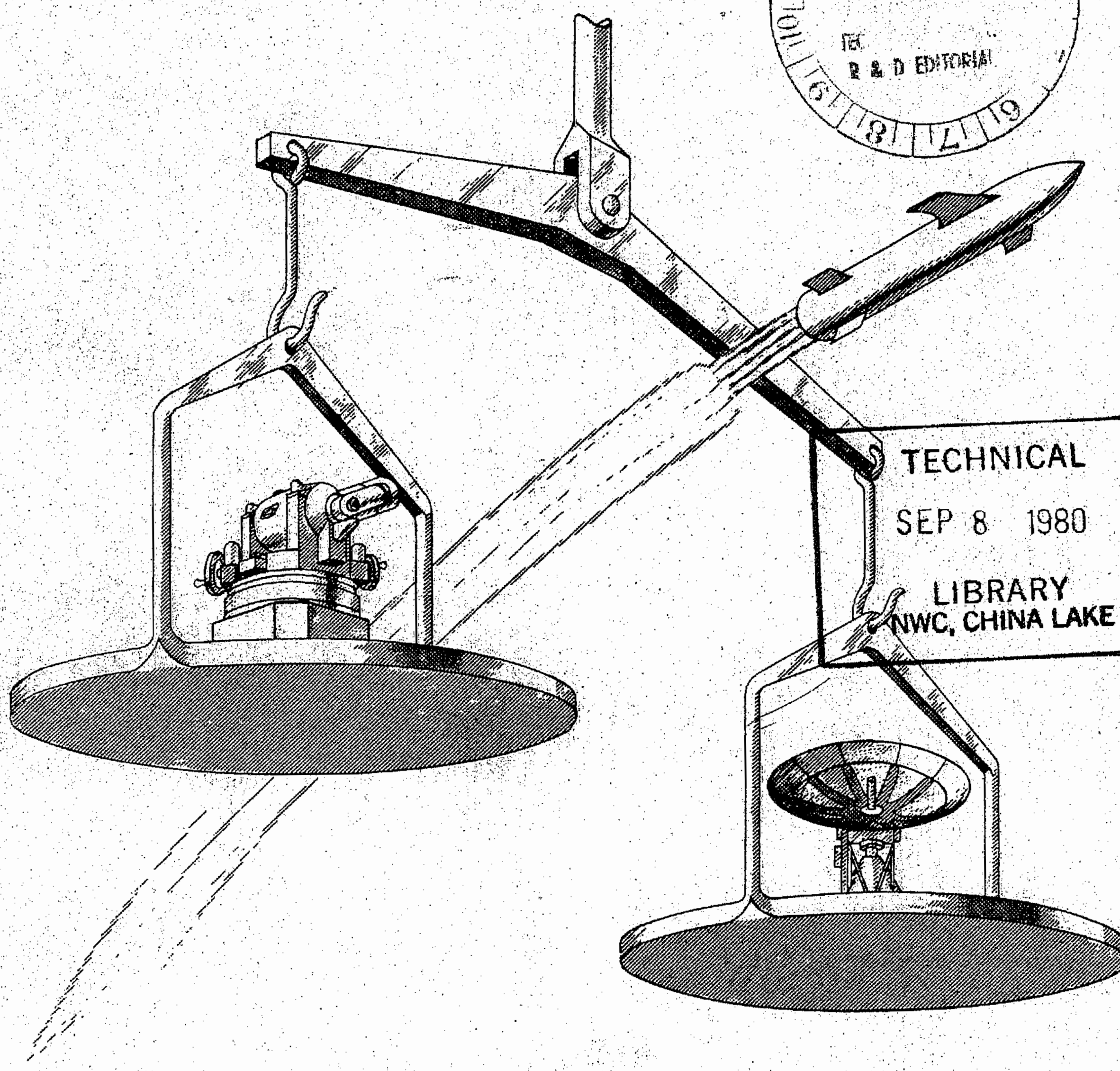

MEASUREMENTS DIVISION

AVIATION ORDNANCE AND TEST DEPARTMENT
U. S. NAVAL ORDNANCE TEST STATION

MISSILE ATTITUDE COMPUTER

M A C

by

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PRELIMINARY DATA

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I. INTRODUCTION

This report describes a preliminary model of an instrument designed to determine the attitude of a missile; that is, the missile's orientation in space at any particular instant. The data necessary for such a determination are the simultaneous observations of at least two tracking stations. These stations provide the azimuth and elevation of the missile. Further measurements on the developed film provide the final piece of data, the angle which the image of the missile makes with the vertical in the plane of the film. With these three pieces of information; namely, the missile's azimuth, elevation and orientation on the photographic film, it is possible for a single operator to ascertain the attitude of a missile in a very short time by use of the instrument described herein.

This analogue computer incorporates the ideas of several researchers. Dr. John Titus first suggested an instrument using only angles to determine the attitude, thereby making a scale model of the range unnecessary. Dr. A. B. Dember, taking up this suggestion, proposed the projection system employed in the instrument. Under the supervision of Mr. A. G. DeBell, Mr. Arnold Zellner constructed a preliminary model which is described in this report. Valuable assistance was rendered by Mr. W. L. Brown in the construction of the model. The following reports by the Optical Measurements Branch, Ballistic Research Laboratories, Aberdeen Proving Ground, provided useful information: OMB Nos. 11, 15, 21 and 695. The first three of these give details about the Optical Stereo Computer and Recorder (Oscar). This device employs astro-compasses, a scale model of the firing range and a mirror system to determine the attitude of a missile. The last report deals with measurements made on a number of A-4 rockets.

II. GENERAL DESCRIPTION

The essential components of this instrument are indicated, as follows, on Photograph No. 1:

(A) An azimuth circle calibrated in degrees or mils.

(B) A model missile (here a cigarette) mounted on a modified astro-compass, whose scales supply the model missile's angles of elevation and azimuth.

(C) Two quadrant arms, calibrated in degrees or mils, of same radius as the horizontal azimuth circle; these radii are measured from center of model missile.

(D) Two projectors mounted on the quadrant arms pointed toward the model missile; these projectors are provided with arrow shaped slit reticles which may be rotated about the optical axis of the projectors to take on an orientation which is the same as that of the missile's image on the film.

III. IMPORTANT FEATURES AND OPERATION

The three principal features of this computer are:

(A) The computer presents an inverted view of actual range conditions. That is, instead of having the observation stations looking up at the missile, the designers decided to have the projectors simulating the cameras shoot beams of light down at the missile. The main advantages of this inversion are simplicity of construction and operation.

(B) The instrument uses only angles in its determination of attitude, eliminating the need for a scale model of the firing range. For example, let tracking cameras No. 1 photograph the missile at azimuth A and elevation E at a particular instant. Similarly, tracking camera No. 2 records azimuth A' and elevation E' at the same instant. This information is usually photographed directly on the film. Quadrant arm No. 1 of the instrument is moved to A degrees on the horizontal azimuth scale and quadrant arm No. 2 is moved to A' degrees to correspond to the azimuths of the tracking cameras. Because of the inversion of the system the horizontal azimuth circle is calibrated counter clockwise (see photograph 1). Then the projector supports are moved up E and E' degrees on their respective quadrant arms to correspond to the elevations recorded by the tracking cameras. Thus, the two projectors have been placed along the line of sight of the tracking cameras. The projectors project thin arrows of light which make angles ϵ and ϵ' with lines tangent to the quadrant arms (see Figure 1).

* The term tracking camera is used here to include both cinetheodolites and tracking telescopes.

These angles are equal to the ones which the image of the missile makes with the vertical on each of the photographic films. The arrow shaped slit reticles are set by means of scales mounted on each projector at the angles $\mathcal{E}$ and $\mathcal{E}'$ which are determined from measurements made on the films of tracking cameras Nos. 1 and 2 respectively.

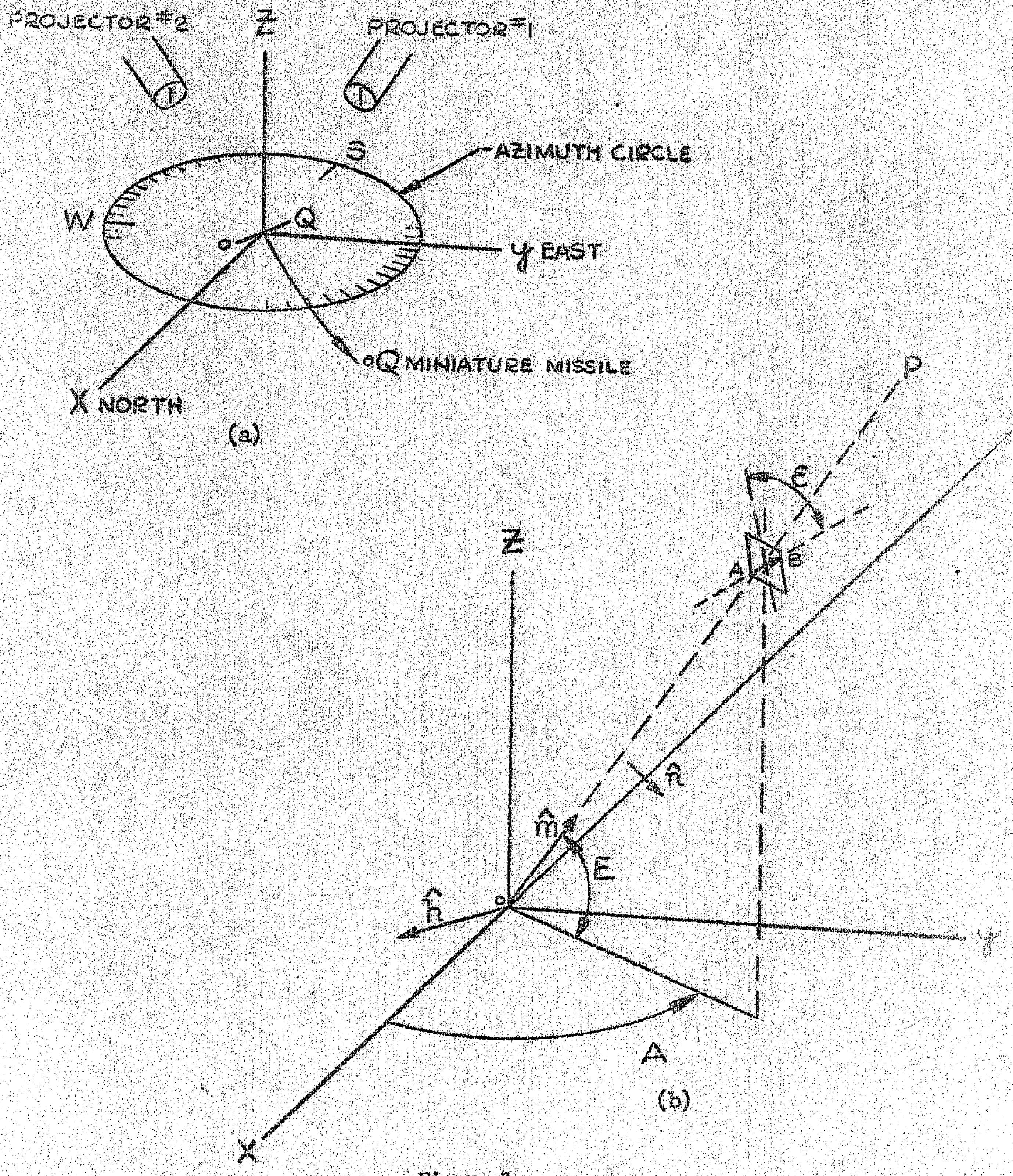
After these settings have been made there are two arrows of light crossed at the center of the model missile. Each arrow lies in a plane defined by the line of sight from tracking camera to missile and the line which is the geometrical projection of the missile's axis in the image plane. The line of intersection of these two planes is the axis of the missile. The model missile must now be oriented so that the two projected arrows are aligned parallel to the missile's axis with the arrowheads pointing in the same direction as the missile head (see Photograph No. 2). This constitutes the final adjustment.

(C) The final adjustment is made by manipulating the controls of the modified astro-compass. This permits the model missile to be oriented conveniently. When the arrows of light from both projectors are parallel to the missile's axis the elevation and azimuth of the missile are read directly from the astro-compass scales.

A simple way to view the system is to consider light traveling from the camera to the missile instead of from the missile to the camera. This reversal of the photographic process makes it evident that the model missile can be oriented to a unique position in which both arrows of light are parallel to the axis of the missile.

IV. MATHEMATICAL ANALYSIS

Orientation of axes with respect to the computer:



(b)

Figure 1

PRELIMINARY DATA

Notation

X - axis Noth)
Y - axis East) All with reference to the azimuth circle
Z - axis overhead) of the computer

$\hat{m}$ = (cos A cos E, sin A cos E, sin E): unit vector along line of sight OP

$\hat{h}$ = (sin A, - cos A, o): unit vector perpendicular to $\hat{m}$ and in horizontal XY plane

$\hat{n}$ = (n_x, n_y, n_z): unit vector perpendicular to $\hat{m}$ and to the image of the axis of the missile (as represented by the arrow AB in figure 1(b).)

A = azimuth setting of arm on instrument (equal to tracking stations azimuth minus 180°)

E = elevation angle

$\mathcal{E}$ = angle which image arrow makes with vertical

The three equations which must be solved to obtain values for n_x, n_y, and n_z are:

$$\hat{h} \cdot \hat{n} = \cos \mathcal{E}; \quad \hat{m} \cdot \hat{n} = 0; \quad \hat{n} \cdot \hat{n} = 1$$

A modification of H. P. Hitchcock's solution* to these equations is:

$$n_x = \cos A \sin E \sin \mathcal{E} + \sin A \cos \mathcal{E}$$

$$n_y = \sin A \sin E \sin \mathcal{E} - \cos A \cos \mathcal{E}$$

$$n_z = -\cos E \sin \mathcal{E}$$

Similarly for the second projector we obtain

$$n'_x = \cos A' \sin E' \sin \mathcal{E}' + \sin A' \cos \mathcal{E}'$$

$$n'_y = \sin A' \sin E' \sin \mathcal{E}' - \cos A' \cos \mathcal{E}'$$

$$n'_z = -\cos E' \sin \mathcal{E}'$$

* — BRL Report No. 695 " Synopsis of Ballistic Measurements of A-4 Rockets" H. P. Hitchcock, Aberdeen Proving Ground, Maryland

Now both $\hat{n}$ and $\hat{n}'$ are perpendicular to the axis of the missile

$\therefore$ $\hat{n}' \times \hat{n} = C\,\hat{p}$ where $\hat{p}$ is a unit vector along axis of missile and C is given by:

$$C = [(n'_y n_z - n'_z n_y)^2 + (n'_z n_x - n'_x n_z)^2 + (n'_x n_y - n'_y n_x)^2]^{1/2}$$

The components of $\hat{p}$ are given by:

$$p_x = \frac{n'_y n_z - n'_z n_y}{C} \quad p_y = \frac{n'_z n_x - n'_x n_z}{C} \quad p_z = \frac{n'_x n_y - n'_y n_x}{C}$$

This provides the direction cosines of the axis of the missile with reference to the chosen coordinate system, as in Figure 2.

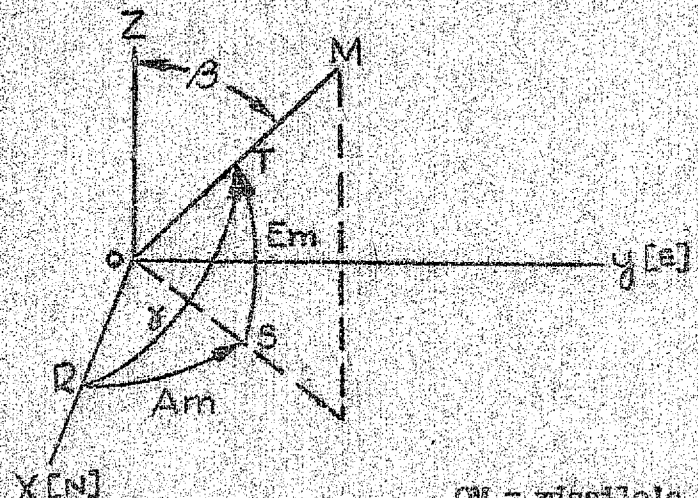


Figure 2

OM = missile's axis
E_m = elevation of missile
A_m = azimuth of missile
$p_z = \cos \beta$
$90^\circ - \beta = E_m$
$p_x = \cos \gamma$

In the spherical triangle RST the sides γ and E_m have been calculated. To obtain A_m, the missile's azimuth apply Napier's rules:

$$\sin(90^\circ - \gamma) = \cos A_m \cos E_m$$

$$\therefore \cos A_m = \frac{\sin(90^\circ - \gamma)}{\cos E_m} \text{ and } A_m \text{ can be computed.}$$

Thus, the attitude has been calculated.

Tracking Camera Data	Settings on MAC	Quadrant Arm No. 1	Quadrant Arm No. 2
Azimuth	Horizontal azimuth	$A = 270^\circ$	$A' = 0^\circ + 180^\circ = 180^\circ$
Elevation	Elevation of Projector	$E = 45^\circ$	$E' = 60^\circ$
Orientation on film	Orientation of arrow	$\varepsilon = 30^\circ$	$\varepsilon' = 330^\circ$

$$n_x = \cos 270^\circ \sin 45^\circ \sin 30^\circ + \sin 270^\circ \cos 30^\circ = -.866$$
$$n_y = \sin 270^\circ \sin 45^\circ \sin 30^\circ - \cos 270^\circ \cos 30^\circ = -.354$$
$$n_z = -\cos 45^\circ \sin 30^\circ = -.354$$
$$n_x' = \cos 180^\circ \sin 60^\circ \sin 330^\circ + \sin 180^\circ \cos 330^\circ = .433$$
$$n_y' = \sin 180^\circ \sin 60^\circ \sin 30^\circ - \cos 180^\circ \cos 330^\circ = .866$$
$$n_z' = -\cos 60^\circ \sin 330^\circ = .433$$

$\hat{n}' \times \hat{n} = C\,\hat{p}$ $\quad\hat{p}$ = unit vector along axis of missile

$$C = [(n_y' n_z - n_z' n_y)^2 + (n_z' n_x - n_x' n_z)^2 + (n_x' n_y - n_y' n_x)^2]^{1/2}$$

$$C = \sqrt{[(.866)(-.354) - (.433)(-.354)]^2 + [(.433)(-.866) - (.433)(-.354)]^2 + [(.433)(-.354) - (.866)(-.866)]^2}$$

$$C = .920$$

$$p_x = \frac{n_y' n_z - n_z' n_y}{C} = \frac{.462}{.920} = .503 \quad \text{then } \delta = 59^\circ$$

$$p_y = \frac{n_z' n_x - n_x' n_z}{C} = -.572$$

$$p_z = \frac{n_x' n_y - n_y' n_x}{C} = .650 \quad \text{then } \beta = 49^\circ$$

Now the elevation of the missile $E_m = 90^\circ - \beta = 90^\circ - 49^\circ = 41^\circ$. Then to find the azimuth we solve the spherical triangle mentioned above.

$$\cos A_m = \frac{\sin (90^\circ - 59^\circ)}{\cos E_m} = .683$$

and $A_m = 47^\circ$

$$\left.\begin{aligned} E_m &= 41^\circ \\ A_m &= 47^\circ \end{aligned}\right\}$$ These two angles describe the orientation (or attitude) of the missile in space

V. ERROR AND SUGGESTED IMPROVEMENTS

The problem calculated above was set up on the preliminary plywood model twice. The results were:

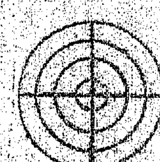
	Azimuth	Elevation
First measurement	49°	41°
Second measurement	48°	41.5°
Calculated value	47°	41°

Another problem calculated in the same manner and then set up on the model yielded the following results:

	Azimuth	Elevation
First measurement	79°	44
Second measurement	79.5°	43°
Third measurement	79°	42°
Calculated value	85°	42°

The accuracy indicated by the above figures is as good as can be expected considering the crude construction of this preliminary model. In a carefully made metal instrument measurements may be made to a quarter of a degree. The principal limitation on the accuracy of the instrument lies in the measurements which must be made of the missile's orientation on the photographic film. This angle, called $\mathcal{E}$ above, will be particularly difficult to measure should the camera happen to photograph the missile either head-on or tail-on. The second case is the more likely to occur. To overcome this difficulty the final model should have arrows shaped as Figure 3(a). Further, the ends of the model missile should appear as shown in Figure 3(b).

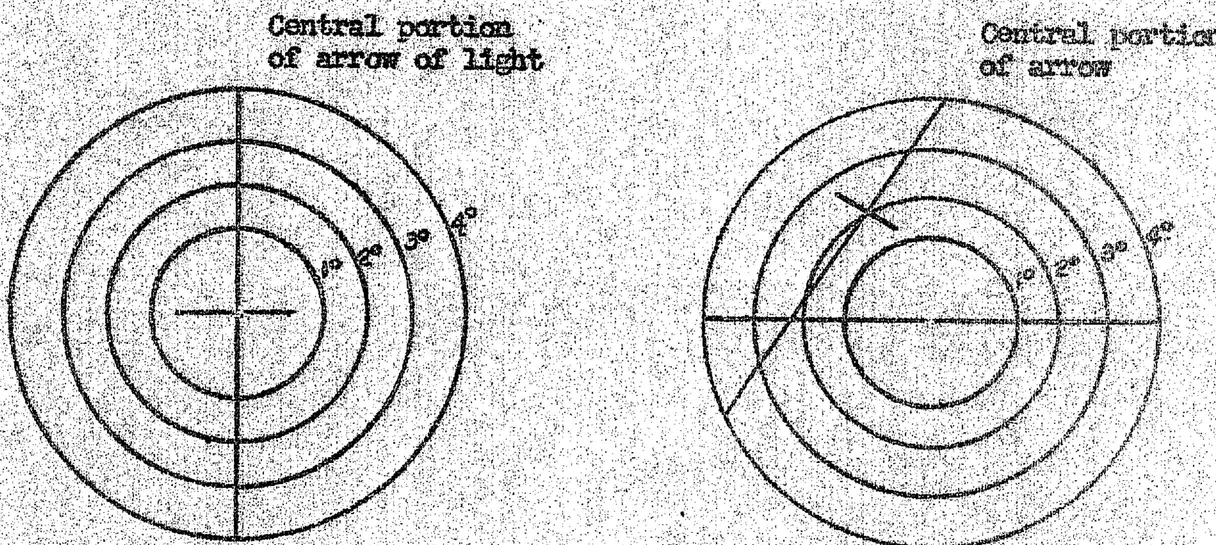
(a)



(b)

Figure 3

The circles on the end of the missile indicate one degree deviation from the exact head-on or tail-on alignments. In the event that one station has a head-on or tail-on image, then the other station's data would be more heavily weighed by paying closer attention to the arrow of light coming from the projector corresponding to the latter station. This arrow is lined up parallel to the axis of the missile by orienting the model missile. Then the intersection of the center of the second arrow of light with circles on the missile's end will directly indicate the deviation from a head-on or tail-on orientation.



(a) Tail-on view

(b) Slight deviation from case on left

Figure 4

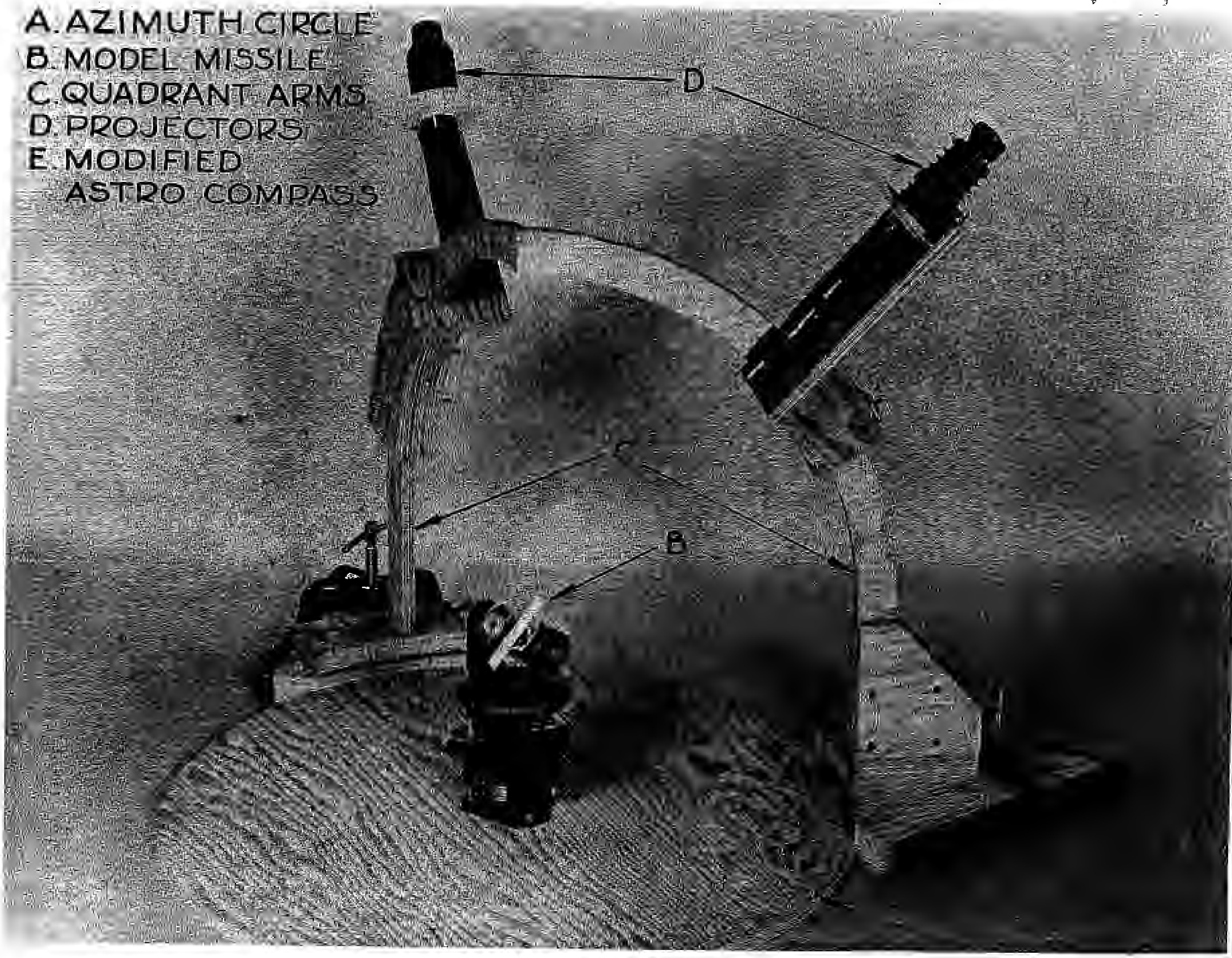
For example, in Figure 4(b) the missile is 2° from a tail-on orientation.

The consistency with which different operators have determined the parallel alignment condition has been very satisfactory, even on this preliminary model. Results of these trials with the quadrant arms set and the projectors fixed yielded the following results:

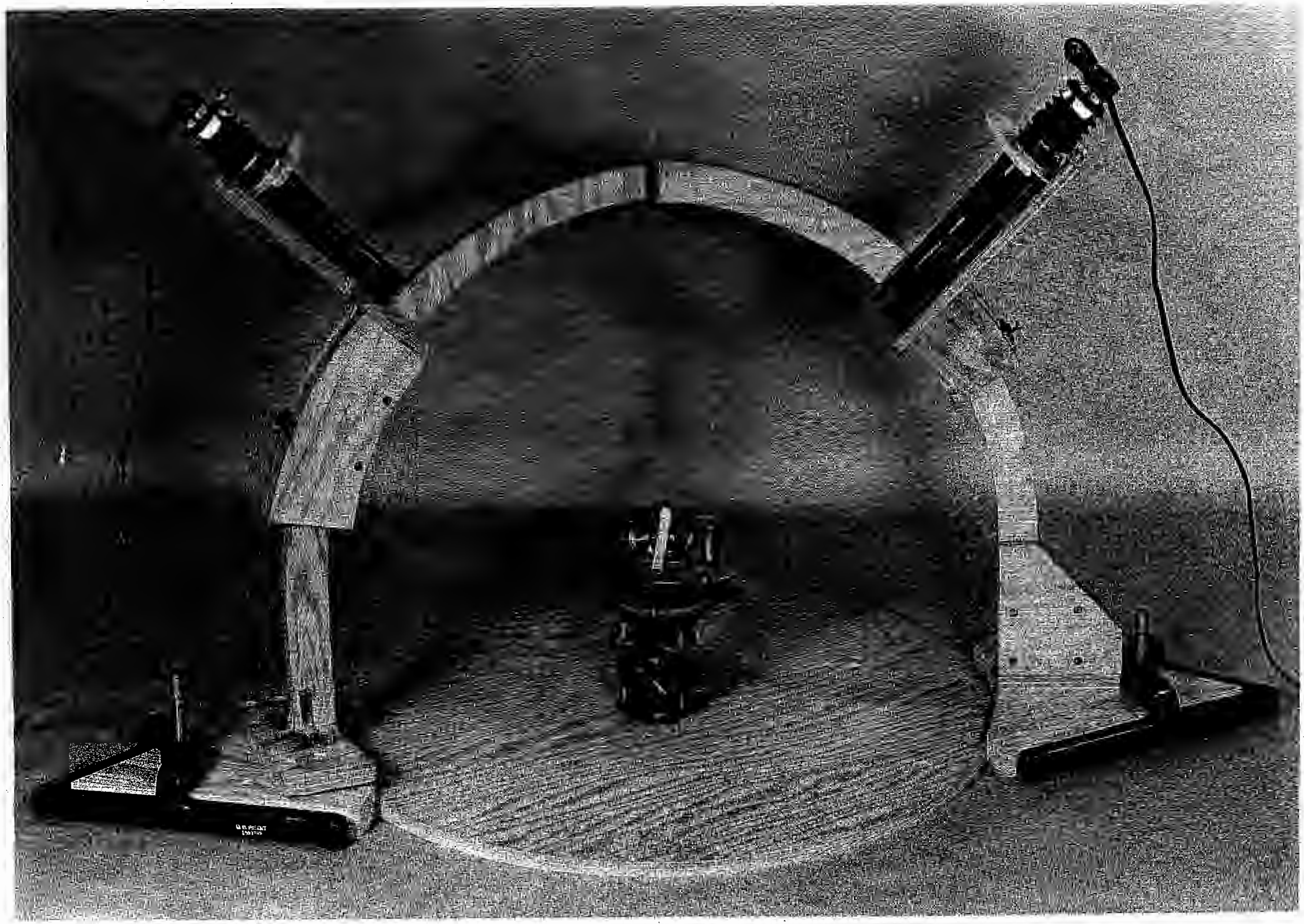
Azimuth	Elevation
50°	31-3/4°
50°	31-1/2°
50°	31-1/4°
50-1/4°	31°

On the final model there will be lines ruled on the surface of the model missile which will serve as a guide in making the parallel alignment. Also, in the final model the missile will be mounted on a simple rod to reduce the possibility of having " blind" positions in the instrument. In this preliminary model there are several positions in which the mount interferes with the arrows of light coming from the projectors.

The use of this analog computer instead of the analytical solution of the equations results in a time saving of at least 90%.



Photograph No. 1 - Preliminary Model of the Missile Attitude Computer



Photograph No. 2 - The Missile Attitude Computer in Operation
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